

# Compositional and dynamic Laffer effects in models with constant returns to scale\*

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## Abstract

There is a renewed interest in the dynamic effects of tax cuts on government revenue. The possibility of tax cuts paying for themselves over time definitely seems like an attractive option for policy makers.

This paper looks at what conditions are required for reductions in capital taxes to be fully self-financing. This is done in a model with constant returns to scale in broad capital. Such a framework exhibits growth; the scope for self-financing tax cuts is therefore different than in the neoclassical growth model, most recently studied by Mankiw and Weinzierl (2006).

Compared to previous literature, I make a methodological contribution in the definition of "Laffer effects" and clarify the role of compositional and dynamic effects in making tax cuts self-financing. I also provide simple analytical expressions for what tax rates are required for tax cuts to be fully self-financing.

The results show that large distortions are required to get Laffer effects. Introducing a labor/leisure choice into the model opens up a new avenue for such effects, however.

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# 1 Introduction

There is a renewed interest in the dynamic effects of tax cuts. This is at least in part due to the recent tax cuts in the US. Methods for not only including "micro" behavioral effects but also dynamic "macro" effects of tax cuts in the US budget process are being discussed (Auerbach, 2006). In a recent paper, Mankiw and Weinzierl provide "back of the envelope" calculations comparing static and dynamic "scoring" for the neoclassical growth model. They argue that tax cuts can, through a new higher steady state level of capital and therefore a larger tax base, to a large extent pay for themselves (Mankiw and Weinzierl, 2006). Leeper and Yang (2007) show that such conclusions can only be drawn with specific assumptions regarding government spending.

This paper follows a different literature than the two papers above and studies effects from tax cuts in a model with constant returns to scale in broad capital. These models are different from the neoclassical growth model studied by Mankiw and Weinzierl. Since they display "endogenous" long-run growth, the scope for dynamic effects is different<sup>1</sup>.

I develop a tractable framework introducing human capital and a labor/leisure choice in the AK-model to make three main points. First, I further define "Laffer effects" in the constant returns models by dividing effects of tax cuts into dynamic and compositional effects. This is crucial when there is more than one factor of production. Second, simple analytical expressions for when tax cuts in AK-style models will fully finance themselves are provided. Third, I follow both the endogenous growth literature and Mankiw and Weinzierl and add a labor/leisure choice to the agent's decision and study how the scope for self-financing tax cuts changes.

Having added leisure to the model, we have a framework with three incentive margins that, as a result of tax cuts, can create Laffer effects on their own or in combination. The three incentive margins are 1) dynamic effects of taxes on interest and growth rates, 2) compositional effects of taxes on production (an "uneven playing field"<sup>2</sup>) and 3) the labor/leisure choice. In a world with the first – dynamic – effect only, there is a direct revenue effect of a tax cut and an indirect effect of different interest and growth rates. The second – compositional – effect comes in when we tax physical and human capital differently; the current tax base is then also affected by tax cuts, adding to the direct revenue effect and the growth effect. Adding the third margin – leisure – there is an additional effect on the tax base through a different labor/leisure choice after a tax cut and there is also an additional growth effect.

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<sup>1</sup>There are thus, broadly speaking, two strands of literature: a neoclassical growth literature and an "endogenous" growth literature. As the neoclassical and endogenous growth models have different long-run properties, the analysis of dynamic effects of tax cuts is also likely to differ.

<sup>2</sup>Goulder and Thalmann (1993) among others use this term to describe the effects of uneven taxation on different types of capital.

In this setup, I derive what combinations of tax rates on physical and human capital are required for a tax cut to be self-financing. The results suggest that dynamic and compositional distortions will need to be large if there are to be Laffer effects; less so, however, if the model contains a labor/leisure choice. I show that the margin opened up by the endogenous labor/leisure choice may be quantitatively important.

Regarding terminology and main scope, this paper follows the tradition of the "endogenous" growth literature and studies "Laffer effects" rather than "dynamic scoring". This means that we are interested in when tax cuts can fully finance themselves, maintaining government spending<sup>3</sup>. I derive conditions for what starting point of tax rates is required for tax cuts to be fully self-financing. As shown by Agell and Persson (2001) and as further detailed here, "maintaining government spending" must be accurately defined and several cases arise. Specifically, I add one definition of Laffer effects to the definitions provided by these authors.

Much of the earlier literature on taxation in "endogenous" growth models has focused on growth effects of taxation in CRTS two-sector models with physical and human capital, e.g. Lucas (1988), King and Rebelo (1990), Rebelo (1991), Pecorino (1993, 1994), Stokey and Rebelo (1995) and Milesi-Ferretti and Roubini (1998a, 1998b). A key aspect of all these papers, as well as of the few studies of Laffer effects, is that in almost all specifications, the return to capital and the growth rate are affected by tax cuts. Milesi-Ferretti and Roubini (1998a, 1998b) clarify the role that different model assumptions have on growth responses from taxation for these two-sector models<sup>4</sup>.

Ireland (1994) and Bruce and Turnovsky (1999) study dynamic Laffer effects in the one-sector AK-model and Novales and Ruiz (2002) in a two-sector model. Agell and Persson (2001) clarify the role of different assumptions regarding "maintaining government spending" in explaining why Ireland and Bruce and Turnovsky get seemingly different results.

This paper extends the study of Laffer effects from the one-sector AK models towards the two-sector models. For this purpose, I add human capital and a

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<sup>3</sup>As the direct revenue effects of tax cuts are negative, government bonds function as the means of intertemporal financing. In the long run, government bonds must obey a transversality condition. This analysis differs from the analysis by Mankiw and Weinzierl (2006) where an atemporal government budget constraint is always obeyed. Their study of "scoring" is therefore different from the study of Laffer effects. There is also a literature, related to both scoring and Laffer effects, comparing the level of present value government revenue along balanced growth paths for different sets of tax rates in calibrated endogenous growth models (Pecorino, 1995; Bianconi, 2000).

<sup>4</sup>Milesi-Ferretti and Roubini (1998a, 1998b) study the balanced growth path responses to taxation in a full catalogue of models that have been used in the literature; they investigate different specifications of leisure, the importance of human capital being a market- (taxed) or home (untaxed) activity and the different cases arising depending on what the human capital production function looks like.

leisure choice to the one-sector AK-framework. For analytical tractability, I first add human capital and work out the effects and then, in a later section, add the leisure decision to the model. The framework has the considerable advantage of there being no transitional dynamics. The economy "jumps" from one growth path to another as a result of tax cuts, thereby facilitating the analysis of Laffer effects<sup>5, 6</sup>.

Section 2 outlines the basic model. Laffer effects are defined in section 3. The conditions for Laffer effects are derived and discussed in section 4. In the model description until section 4, the first two effects of taxation, the dynamic effect and the compositional effect from above, are present in the analysis. Section 5 introduces the third effect of taxation by endogenizing the leisure decision and shows how this additional incentive margin affects the scope for Laffer effects. Section 6 summarizes and discusses the results. A list of all variables and parameters are included in the appendix. In the appendix, the relationship between the model to the general two-sector model with human and physical capital is also discussed. Finally, the level of leisure in a special case is solved for.

## 2 The model

I set up a perfect foresight and full commitment model with utility maximizing agents holding physical and human capital. Agents derive utility from consumption. The capital is rented out to firms and agents pay tax on the returns to their capital stocks. The government uses tax receipts to finance lump-sum transfers and government consumption. Having set up the model, I define Laffer effects in section 3 and in section 4 then ask: what combination of tax rates is required for the government to be able to reduce a tax rate but still maintain its spending paths?

### 2.1 Production and capital

The model used in this paper is a modified AK-model, a one-sector model with physical and human capital in the production function. It has constant returns

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<sup>5</sup>In order to analytically isolate the three incentive margins I impose (1) the restriction of one common production function for physical and human capital and (2) no restriction on deinvestment in either type of capital. The assumptions imply that a two-sector model with equal production functions for physical and human capital collapses into the one-sector model presented here. There will be no transitional dynamics since adjustments to tax changes are immediate. There are growth effects, though.

<sup>6</sup>Novales and Ruiz (2002) parametrize a version of the two-sector model with physical and human capital and use numerical methods to study Laffer effects.

to scale in physical capital  $K$  and human capital  $H$  altogether. The production function for physical as well as human capital is  $F(K, H) = AK^\alpha H^{1-\alpha}$ , i.e. Cobb-Douglas<sup>7</sup> with  $0 < \alpha < 1$ .

Output in the economy is used for consumption or for building physical and human capital stocks. It is then assumed that output can be directly used for both physical capital build-up and human capital build-up and that one unit of physical capital can be converted into one unit of human capital. It is also assumed that investment in physical capital and human capital can be negative and immediate. This implies that capital stocks can "jump" from one level to another; the aggregate of physical and human capital cannot jump, however. In effect, there is thus one aggregate capital stock, defined in per-capita terms as  $z_t = k_t + h_t$ , where  $t$  is a time index<sup>8</sup>.

## 2.2 Representative agent optimization

Agents derive utility from consumption and have an infinite time horizon. Utility maximizing agents sell their physical and human capital to profit maximizing firms and receive factor returns. The agent also receives a lump-sum transfer from the government and returns from government bonds. Income is spent on consumption or invested in the assets of the economy, physical and human capital and government bonds. Income is also used to pay taxes on the returns on these assets. The government uses tax receipts to finance government consumption and lump-sum transfers to the agents. Depreciation rates are set at zero and there is no population growth.

Before solving the representative agent's optimal consumption path, we derive his return to capital. Firms rent physical and human capital from the agents in order to maximize their profits with respect to inputs  $k_t$  and  $h_t$ . The per-capita production function is  $f(k_t, h_t) = Ak_t^\alpha h_t^{1-\alpha}$ . From the competitive equilibrium condition that  $r_t = \partial f / \partial k_t$  and  $w_t = \partial f / \partial h_t$ , where  $r_t$  is the return on physical capital and  $w_t$  is the return on human capital, the standard arbitrage condition of equal after-tax returns on  $k_t$  and  $h_t$  becomes

$$r_t(1 - \tau_k) = w_t(1 - \tau_h), \quad (1)$$

where  $\tau_k$  and  $\tau_h$  are taxes on returns to physical and human capital, respectively.

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<sup>7</sup>Stokey and Rebelo (1995) use the more general CES production function studying growth effects from tax rates and conclude that the elasticities of substitution in production are relatively unimportant.

<sup>8</sup>Our model is equivalent to a two-sector model with equal production functions for physical and human capital and no restrictions on deinvestment of  $k$  and  $h$ . As a result of our assumptions, we get a framework where responses to tax cuts are immediate. Absent transitional dynamics between the old and new growth paths, we are then able to decompose the effects of tax cuts into compositional and dynamic effects. In the appendix, we discuss the relation between our model and the two-sector models. See Barro and Sala-i-Martin (1995) for a presentation of the model used here.

Condition (1) allows us to define the agent's after tax return to capital ( $k, h$  as well as  $z$ ) to become

$$\phi \equiv r_t(1 - \tau_k) = A\alpha^\alpha(1 - \alpha)^{1-\alpha}(1 - \tau_k)^\alpha(1 - \tau_h)^{1-\alpha}. \quad (2)$$

This return  $\phi$  will also be the return paid by the government on the stock of government bonds, defined as  $b_t$ <sup>9</sup>. The agent's total wealth, defined as  $W_t \equiv z_t + b_t$ , thus earns the return  $\phi$ .

The agent maximizes lifetime utility from consumption subject to the budget constraint, i.e.

$$\begin{aligned} \max \int_0^\infty U(c_t, G_t) e^{-\rho t} dt \quad \text{s.t.} \\ \dot{W}_t = \phi W_t + T_t - c_t, \quad \text{where} \quad W_0 = z_0 + b_0. \end{aligned} \quad (3)$$

There is also a transversality condition.  $U(c_t, G_t)$  is the instantaneous utility function,  $c_t$  is private consumption,  $G_t$  is government consumption,  $\rho$  is the time preference factor of the agent,  $T_t$  are lump-sum transfers received from the government,  $z_0$  is period-zero total capital and  $b_0$  is period-zero government bonds<sup>10</sup>. Dotted variables are time derivatives. Time indices will normally be suppressed. The utility function is additively separable in private consumption  $c$  and government consumption  $G$ ,  $U(c, G) = u(c) + v(G)$  where  $u(c)$  takes the Constant Inter-temporal Elasticity of Substitution (CIES) form

$$u(c) = \frac{c^{1-\theta}}{1-\theta}, \quad (4)$$

where  $\theta$  is the inverse of the intertemporal elasticity of substitution. Attaching the dynamic Lagrange multiplier  $\lambda$  to the budget constraint in (3), using control  $c$  and state  $W$ , the first-order conditions of this problem are

$$u'(c) e^{-\rho t} = \lambda \quad (5)$$

$$\lambda \dot{\phi} = -\dot{\lambda} \quad (6)$$

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<sup>9</sup>The return from bonds is taxed with the physical capital tax  $\tau_k$ , so that the pre-tax return to bonds  $R_t$  would be determined by  $\phi = R_t(1 - \tau_k)$ .

<sup>10</sup>Regarding notation, we use capital letters for government spending variables, i.e. transfers  $T$  and government consumption  $G$ . We use  $W$  for per-capita wealth in order to not confuse it with the return to human capital,  $w$ . Small letters are used for all other stocks and flows. We use the capital letter  $A$  in the production function because of the resemblance with the AK-model.

and the transversality condition is  $\lim_{t \rightarrow \infty} \lambda_t W_t = 0$ .

Using the CIES utility function and conditions (5-6) gives the Euler equation

$$\gamma \equiv \frac{\dot{c}}{c} = \frac{1}{\theta} (\phi - \rho). \quad (7)$$

The growth rate of consumption  $\gamma$  depends on the degree of intertemporal substitution  $\sigma \equiv \theta^{-1}$ , the after-tax return on capital  $\phi$  (from 2) and the time preference factor  $\rho$ . The degree of intertemporal substitution has the usual interpretation: an agent with a low degree of intertemporal substitution prefers a stable consumption path and will not react to tax changes to any considerable extent. A low after-tax return  $\phi$  will discourage investment and slow growth. So will a high degree of impatience ( $\rho$  high) of the agent, because future consumption flows are less valued. Taxes on physical and human capital affect the growth rate, through  $\phi$ , to the same degree as physical and human capital affect total output. Reductions in  $\tau_k$  and  $\tau_h$  make investment more productive and hence increase the growth rate,  $\partial\gamma/\partial\tau_k < 0$  and  $\partial\gamma/\partial\tau_h < 0$ .<sup>11</sup>

### 2.3 Composition effect, return to capital and total production

Since we will work with compositional as well as dynamic effects from tax cuts, we need to keep track of how tax changes affect the composition of the human-to-physical capital stocks and how this affects production and returns to capital. This section therefore discusses three variables that will be important in what follows: the agent's  $h/k$ -ratio, the private return to capital  $\phi$  (from above) and the economy-wide return to capital.

From the above, we get the equilibrium  $h/k$ -ratio, derived from the arbitrage condition in (1),

$$\frac{h}{k} \equiv \Omega = \frac{1 - \alpha}{\alpha} \frac{1 - \tau_h}{1 - \tau_k}. \quad (8)$$

$\Omega$  is the ratio between  $h$  and  $k$  for an optimizing agent. A lower  $\alpha$  makes  $k$  less important in production increasing the  $h$ -to- $k$  ratio. An increase in  $\tau_k$  has the same effect<sup>12</sup>.

<sup>11</sup>The transversality condition  $\lim_{t \rightarrow \infty} \lambda_t W_t = 0$  implies that the consumption growth rate must be smaller than the private return on capital;  $\phi - \gamma > 0$ . For values of  $\theta$  above or equal to unity, this condition is always satisfied. Below unity we require  $A\alpha^\alpha(1-\alpha)^{1-\alpha}(1-\tau_k)^\alpha(1-\tau_h)^{1-\alpha} < (1-\theta)^{-1}\rho$ .

<sup>12</sup>As seen in (2), a higher  $\tau_k$  reduces  $\phi$  with the factor  $(1-\tau_k)^\alpha$  and not  $(1-\tau_k)$ . Since  $h/k$  increases, physical capital  $k$  becomes more scarce and its return  $r$  goes up, partly but not fully compensating the effect of the tax cut on the private return and growth.

In interpreting  $\Omega$ , note that maximizing output  $Ak^\alpha h^{1-\alpha}$  subject to the constraint  $h + k = z$  would yield a ratio  $h/k = (1 - \alpha)/\alpha$ . The agent's  $h/k$  ratio and therefore what is used for production differs from this value as soon as  $\tau_h \neq \tau_k$ . A differentiated tax treatment of  $h$  and  $k$  thus adds the second effect discussed in the introduction, a compositional distortion in production, to the first effect, the dynamic distortion always present when capital is taxed. This compositional distortion from a differentiated capital taxation is important in the analysis of Laffer effects. We can see this importance by defining the economy-wide return to capital which, using  $h/k = \Omega$ , is

$$\beta \equiv \frac{wh + rk}{h + k} = \frac{A\Omega^{1-\alpha}}{1 + \Omega} = \frac{A\alpha^\alpha(1 - \alpha)^{1-\alpha}(1 - \tau_k)^\alpha(1 - \tau_h)^{1-\alpha}}{\alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_h)}. \quad (9)$$

It can be shown that the economy-wide return to capital  $\beta$ , with one tax rate given, is at its maximum when the second tax is set equal to the first tax. Otherwise,  $\beta$  is below its maximum value.  $\beta$  is nothing else than the de facto production factor in the economy which we see by rewriting  $Ak^\alpha h^{1-\alpha}$  as follows:

$$Ak^\alpha h^{1-\alpha} = \beta z. \quad (10)$$

Total production is therefore tax-dependent, a feature which is absent in standard one-sector AK-models. An uneven taxation means a suboptimal use of resources and therefore lower production. Consequently, a reduction in the highest tax will decrease this distortion and production will jump to a higher level, which will open up a new margin for Laffer effects<sup>13</sup>.

For future reference, also note that

$$\phi = \beta(1 - \tau_{avg}), \quad (11)$$

where the "average tax rate"  $\tau_{avg} \equiv \alpha\tau_k + (1 - \alpha)\tau_h$  has been used. Whereas  $\beta$  is the economy-wide return to capital, the agents face the (lower) private return  $\phi$  because taxes must be paid. Without taxes, the returns are equal<sup>14</sup>.

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<sup>13</sup>In our framework, there are no traditional dynamics but immediate adjustment in the  $h/k$ -ratio as a result of tax cuts. If we had a more general model with different production functions for physical and human capital, we would get transitional dynamics as a result of tax cuts but still, along a balanced growth path, a constant  $h/k$ -ratio. There is still a compositional reoptimization as a response to tax cuts. It would not be immediate, however. As will be discussed in detail in the following section, the tax cuts that will be considered are such that we reduce the highest tax and hence, increase GDP.

<sup>14</sup>We use the term "economy-wide" to refer to the pre-tax return to capital in the economy.



## 2.4 Intertemporal constraints

Throughout the analysis of Laffer effects, the tools of analysis will be the consumption rule and the present value resource constraint. These will describe how the representative agent responds to tax changes and, as a result, what scope there is for Laffer effects. The consumption rule is derived by integrating the budget constraint

$$\dot{W} = \phi W + T - c. \quad (12)$$

The present value budget constraint, using initial total wealth  $W_0 = z_0 + b_0$  and  $c_t = c_0 e^{\gamma t}$  from (7) and applying the transversality condition, becomes

$$\frac{c_0}{\phi - \gamma} = z_0 + b_0 + \int_0^\infty T_t e^{-\phi t} dt, \quad (13)$$

where  $c_0$  is period-zero private consumption. This relationship says that the present value of consumption should be equal to initial assets plus the present value of transfers received from the government. We get the consumption rule by multiplying through by  $(\phi - \gamma)$ ,

$$c_0 = (\phi - \gamma) \left( z_0 + b_0 + \int_0^\infty T_t e^{-\phi t} dt \right). \quad (14)$$

This consumption rule, which depends both on the transfer and the tax policy of the government, will be used to study how government consumption can vary with tax rates complying with the economy's resource constraint. In the resource constraint, the production of the economy is either consumed by the government or by the agents or added to the stocks of  $k$  and  $h$ ,  $Ak^\alpha h^{1-\alpha} = c + G + \dot{k} + \dot{h}$ . With GDP written as  $\beta z$  (from 10), the resource constraint becomes

$$\dot{z} = \beta z - c - G, \quad (15)$$

and the present value resource constraint, using initial capital  $z_0$  and  $c_t = c_0 e^{\gamma t}$  and applying the transversality condition, becomes

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It is different, however, from the first-best (social) return as long as taxes are differentiated. In the AK-model, the private return would be  $A(1 - \tau_k)$  and the economy-wide (and social) return is  $A$ . Expression (11) is the same relationship in a model with two types of capital and two tax rates.

$$\int_0^\infty G_t e^{-\beta t} dt + \frac{c_0}{\beta - \gamma} = z_0. \quad (16)$$

The constraint says that the present value of total consumption must equal initial resources. Note that it is the "economy-wide" discount rate  $\beta$  which is of importance for the use of resources, whereas it is the private return  $\phi$  that determines the behavior of the agent in the consumption rule.

We have now derived the tools to study Laffer effects, i.e. the tools for studying how government spending reacts to tax cuts and if it will be possible to "maintain government spending" after tax cuts. In the following sections, different versions of expressions (14) and (16) will be differentiated with respect to tax rates in order to study Laffer effects. Before that, however, we need to rigorously define Laffer effects and what we mean with "maintaining government spending".

### 3 Definitions of Laffer effects

The definitions of Laffer effects follow and extend the work by Agell and Persson (2001) where differences between earlier results on dynamic Laffer effects were clarified. It extends this work to a context where there is more than one factor of production and therefore not only dynamic but also compositional effects of taxes. Precisely because the model contains compositional as well as dynamic distortions, I use the term "Laffer effect" instead of "dynamic Laffer effect".

Three definitions of Laffer effects are presented. Definition 1 follows the Agell and Persson definition whereas definitions 2 and 3 comprise two slightly different cases that collapse into one case in the basic AK framework. The difference between the three definitions is related to what we mean by "maintaining government spending". In the model presented so far, a balanced growth path exists where private consumption, capital stocks and government consumption and transfers all grow at the same rate. After a tax cut, the return to private capital  $\phi$  and the growth rate of consumption  $\gamma$  increase. We can then either allow for government consumption and transfers to adjust their growth rate to the new higher rate or they can maintain their pre tax cut growth rates.

We also need to distinguish between the case where we account for the jump in production as a result of tax cuts and the case where we do not. After a tax cut, because the "economy-wide" return  $\beta$  is tax-dependent, period-zero GDP discretely adjusts from  $f_0(k_0^{\text{pre}}, h_0^{\text{pre}}) \equiv \beta^{\text{pre}} z_0$  to  $f_0(k_0^{\text{post}}, h_0^{\text{post}}) \equiv \beta^{\text{post}} z_0$ .

Definition 2 does not take this discrete adjustment into consideration whereas definition 3 does. That is, in definition 2, because GDP discretely increases as a result of a tax cut, the transfer-to-GDP ratio goes down for a given period-zero transfer  $T_0$  and we allow this to happen. This is why we will use the post tax cut GDP  $f_0(k_0^{\text{post}}, h_0^{\text{post}})$  in the transfer-to-GDP ratio in definition 2 below. We do not require  $T_0$  to adjust to maintain the original ratio. In definition 3, we want to maintain the original transfer-to-GDP ratio and therefore divide by the original GDP,  $f_0(k_0^{\text{pre}}, h_0^{\text{pre}})$ . In the basic AK framework, definitions 2 and 3 collapse into one case only<sup>15</sup>.

I now state the three definitions and then clarify with an example how they differ.

**Definition 1** Assume that the resource constraint  $\int_0^\infty G_t e^{-\beta t} dt + c_0 (\beta - \gamma)^{-1} = z_0$  holds for some initial tax rates  $\tau_k^{\text{pre}}$  and  $\tau_h^{\text{pre}}$  and flows of government consumption  $(G_t)_0^\infty$  and transfers  $(T_t)_0^\infty$ . If there is some lower set of tax rates  $\tau_k^{\text{post}} \leq \tau_k^{\text{pre}}$  and  $\tau_h^{\text{post}} \leq \tau_h^{\text{pre}}$ , where at least one of the inequalities should be strict, that allows the government to maintain its transfer program  $(T_t)_0^\infty$  and for some time  $\Delta t > 0$  increase its consumption flow  $G_t$  and not decreasing it at any other time, there is a Laffer effect.

In definition 1, government transfers  $T_t$  follow their pre tax cut path, even after the tax cut. This path of  $T_t$  will be taken to be  $T_t = T_0^{\text{pre}} e^{\gamma^{\text{pre}} t}$  where  $\gamma^{\text{pre}}$  is the pre-tax cut growth rate of consumption, GDP and capital stocks and  $T_0^{\text{pre}}$  constitute the pre tax cut period-zero level of transfers. When implementing definition 1 in this paper, government consumption  $G_t$  will also grow at the old growth rate of private consumption,  $\gamma^{\text{pre}}$ , and we will ask the question whether period-zero government consumption  $G_0$  can increase when a tax is reduced<sup>16</sup>.

**Definition 2** Assume that the resource constraint  $\int_0^\infty G_t e^{-\beta t} dt + c_0 (\beta - \gamma)^{-1} = z_0$  holds for some initial tax rates  $\tau_k^{\text{pre}}$  and  $\tau_h^{\text{pre}}$  and flows of government consumption  $(G_t)_0^\infty$  and transfers  $(T_t)_0^\infty$ . If there is some lower set of tax rates  $\tau_k^{\text{post}} \leq \tau_k^{\text{pre}}$  and  $\tau_h^{\text{post}} \leq \tau_h^{\text{pre}}$ , where at least one of the inequalities should be strict, that allows the government to maintain its transfer to GDP ratio, i.e. at all times after the tax cut keep  $T_t/f_t(k, h) = T_0^{\text{pre}}/f_0(k_0^{\text{post}}, h_0^{\text{post}})$ , and for some time  $\Delta t > 0$  increase its consumption to GDP ratio  $G_t/f_t(k, h)$  to exceed  $G_0^{\text{pre}}/f_0(k_0^{\text{post}}, h_0^{\text{post}})$  and not decreasing it at any other time, there is a Laffer effect.

<sup>15</sup>Regarding notation, the superindices "pre" and "post" refer to the values pre- and post-tax cut, respectively. The subindex  $t$  refers to time, a subindex 0 therefore means the value at time zero.

<sup>16</sup>That is, can  $G_t$ , as a response to a tax cut, shift up to a higher level and then continue to grow at its old growth rate but starting at this new higher level so that  $G_t$  is permanently on a higher level than before the tax cut?

Using the more demanding definition 2, all government spending follows the new higher growth rate of private consumption and GDP, even after the tax cut. That is,  $T_t = T_0^{\text{pre}} e^{\gamma^{\text{post}} t}$  and we will let  $G_t$  grow at the rate  $\gamma^{\text{post}}$  as well and we ask the question whether period zero government consumption  $G_0$  can increase when a tax is reduced.

**Definition 3** Assume that the resource constraint  $\int_0^\infty G_t e^{-\beta t} dt + c_0 (\beta - \gamma)^{-1} = z_0$  holds for some initial tax rates  $\tau_k^{\text{pre}}$  and  $\tau_h^{\text{pre}}$  and flows of government consumption  $(G_t)_0^\infty$  and transfers  $(T_t)_0^\infty$ . If there is some lower set of tax rates  $\tau_k^{\text{post}} \leq \tau_k^{\text{pre}}$  and  $\tau_h^{\text{post}} \leq \tau_h^{\text{pre}}$ , where at least one of the inequalities should be strict, that allows the government to maintain its initial transfer to GDP ratio at all times, i.e. keep  $T_t/f_t(k, h) = T_0^{\text{pre}}/f_0(k_0^{\text{pre}}, h_0^{\text{pre}})$ , and for some time  $\Delta t > 0$  increase its consumption to GDP ratio  $G_t/f_t(k, h)$  to exceed  $G_0^{\text{pre}}/f_0(k_0^{\text{pre}}, h_0^{\text{pre}})$  and not decreasing it at any other time, there is a Laffer effect.

Using the even more demanding definition 3, all government spending follows the new higher growth rate of private consumption,  $\gamma^{\text{post}}$ . In addition, the new period zero transfers  $T_0^{\text{post}}$  and therefore the whole path of transfers  $T_t$  has made a discrete adjustment to match the discrete adjustment in GDP, i.e.  $T_0^{\text{post}}/T_0^{\text{pre}} = f_0(k_0^{\text{post}}, h_0^{\text{post}})/f_0(k_0^{\text{pre}}, h_0^{\text{pre}})$ . We ask the question whether period zero government consumption  $G_0$  can make a discrete adjustment that is larger than the adjustment in GDP and then grow at the rate  $\gamma^{\text{post}}$ .

To further clarify the difference between the three definitions, imagine a tax cut that increases the consumption growth rate from 2% to 3% and as a result of the tax cut, GDP experiences a 1% discrete jump from 1.00 to 1.01 in period zero. With definition 1, transfers  $T_t$  should not jump in period zero and continue to grow with 2% and we ask whether period-zero  $G_t$  can jump to a higher level and then grow with 2%.

With definition 2, transfers  $T_t$  should also not jump in period zero but then grow with 3% and we ask whether period-zero  $G_t$  can jump to a higher level and then grow with 3%.

With definition 3, transfers  $T_t$  should jump up 1% in period zero and then grow with 3% and we ask whether period-zero  $G_t$  can jump more than 1% and then grow with 3%.

In the following section, I derive analytical conditions for when Laffer effects of definitions 1 and 2 occur, interpret the results and show that definition 3 effects can never occur<sup>17</sup>.

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<sup>17</sup>Ireland (1994) and Novales and Ruiz (2002) use definition 1 in characterizing Laffer effects.

## 4 Conditions to get Laffer effects

### 4.1 Pre tax cut setting

In order to study Laffer effects, we start out in a situation at time  $t = 0$  with initial capital  $z_0$ , zero outstanding government debt ( $b_0 = 0$ ) and government transfers and consumption equalling government revenue,  $T_0 + G_0 = r^{\text{pre}}\tau_k^{\text{pre}}k_0^{\text{pre}} + w^{\text{pre}}\tau_h^{\text{pre}}h_0^{\text{pre}}$ . Using the equilibrium expressions for  $r, w$  and  $h/k$ , this expression can be written as

$$T_0 + G_0 = \tau_{\text{avg}}^{\text{pre}}\beta^{\text{pre}}z_0. \quad (17)$$

Prior to a tax cut GDP, private consumption, capital stocks as well as government consumption and transfers all grow at the pre tax cut growth rate  $\gamma^{\text{pre}} = \frac{1}{\theta}(\phi^{\text{pre}} - \rho)$ . Taxes are then changed according to  $\tau_k^{\text{post}} \leq \tau_k^{\text{pre}}$  and  $\tau_h^{\text{post}} \leq \tau_h^{\text{pre}}$ , where at least one of the inequalities should be strict. For the analytical expressions derived below, we restrict ourselves to reduce one tax at a time, i.e. we either have  $(\tau_k^{\text{post}} < \tau_k^{\text{pre}}, \tau_h^{\text{post}} = \tau_h^{\text{pre}})$  or  $(\tau_k^{\text{post}} = \tau_k^{\text{pre}}, \tau_h^{\text{post}} < \tau_h^{\text{pre}})$ . Moreover, we are naturally interested in decreasing the highest tax as this reduces the compositional distortion in production.

For the subsequent analysis, it will be useful to express the tax-derivatives of the consumption growth rate  $\gamma$  and the economy-wide return to capital  $\beta$  as functions of the tax-derivatives of the private return to capital  $\phi$ . A few algebraic steps will show that

$$\frac{\partial \gamma}{\partial \tau_h} = \frac{1}{\theta} \frac{\partial \phi}{\partial \tau_h} \text{ and } \frac{\partial \beta}{\partial \tau_h} = \Gamma_h \frac{\partial \phi}{\partial \tau_h} \text{ where } \Gamma_h = \frac{\alpha(\tau_h - \tau_k)}{(1 - \tau_{\text{avg}})^2} \quad (18)$$

$$\frac{\partial \gamma}{\partial \tau_k} = \frac{1}{\theta} \frac{\partial \phi}{\partial \tau_k} \text{ and } \frac{\partial \beta}{\partial \tau_k} = \Gamma_k \frac{\partial \phi}{\partial \tau_k} \text{ where } \Gamma_k = \frac{(1 - \alpha)(\tau_k - \tau_h)}{(1 - \tau_{\text{avg}})^2}. \quad (19)$$

$\Gamma_h$  and  $\Gamma_k$  are important factors in the Laffer effect analysis. They represent the second effect of taxation in the model, the impact of the compositional distortion from a differentiated tax treatment of  $h$  and  $k$ . In (18) and (19), we get that if  $\tau_h = \tau_k$ , both  $\Gamma_h$  and  $\Gamma_k$  are zero and  $\beta$  is not tax dependent. A non-zero value of either  $\Gamma_h$  or  $\Gamma_k$  means that we have a compositional distortion and that the maximum production capacity is not achieved (as GDP equals  $\beta z$  from 10). As discussed earlier, tax changes that affect  $\beta$  will therefore make available more/less resources in all periods and affect the possibility for Laffer effects (more resources when we reduce the highest tax which is what we are

interested in). For future reference, we also note that if  $\Gamma_h$  or  $\Gamma_k$  are larger than unity, we get a larger change in the economy-wide return  $\beta$  than in the private return to capital  $\phi$  when taxes are changed.

## 4.2 Mathematical criterion

We study under what conditions tax cuts give rise to Laffer effects. Let subindex  $i$  refer to either the physical capital or human capital tax. The criterion to get a Laffer effect is  $\partial G_0 / \partial \tau_i < 0$  for definitions 1 and 2 Laffer effects and  $\partial (G_0 / \text{GDP}) / \partial \tau_i < 0$  for definition 3 Laffer effects.

We now derive what conditions must be fulfilled in order to get Laffer effects according to definitions 1-2 and then interpret the results, specifically discussing the compositional effect that  $\Gamma_h$  and  $\Gamma_k$  represent. We also show that definition 3 effects are not possible. Since definition 1 Laffer effects are very similar to the analysis in Agell and Persson (2001), the reader is referred to these authors for a full discussion.

## 4.3 Laffer effect according to definition 1

Following definition 1, we will let  $T$  grow at its pre tax cut growth rate  $\gamma^{\text{pre}}$  and study scope for increased  $G$ .  $G$  is set to grow at the original growth rate  $\gamma^{\text{pre}}$  as well and it is therefore enough to study the impact on  $G_0$ ,  $G$  in period zero. The consumption rule and the present value resource constraint, expressions (14) and (16), are repeated with these assumptions for  $T$  and  $G$ :

$$\frac{c_0}{\phi - \gamma} = z_0 + \frac{T_0}{\phi - \gamma^{\text{pre}}} \quad (20)$$

$$\frac{G_0}{\beta - \gamma^{\text{pre}}} = z_0 - \frac{c_0}{\beta - \gamma}. \quad (21)$$

We differentiate (20) and (21) with respect to either of the tax rates and study whether such a tax change makes the new  $G_0$  comply with the condition for a Laffer effect,  $\partial G_0 / \partial \tau_i < 0$ . A change in taxation will, through its effect on growth, the private discount rate and future value of transfers in the first constraint affect  $c_0$ . This change in  $c_0$  then adds to the effects on the growth rate and on the economy-wide discount rate in the second constraint to give a total effect on  $G_0$  such that the resource constraint is always fulfilled<sup>18</sup>. Note

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<sup>18</sup>That is;  $G_0$  is residually calculated such that the resource constraint always holds.

that  $\partial/\partial\tau_i$  can mean a change in either tax rate,  $\tau_h$  or  $\tau_k$ . Differentiation of (21) and (20) gives<sup>19</sup>

$$\frac{\partial G_0}{\partial\tau_i} = -\frac{\partial c_0}{\partial\tau_i} - \frac{c_0}{\beta - \gamma} \frac{\partial\gamma}{\partial\tau_i} + z_0 \frac{\partial\beta}{\partial\tau_i}, \quad (22)$$

where

$$\frac{\partial c_0}{\partial\tau_i} = \frac{\partial(\phi - \gamma)}{\partial\tau_i} \left( z_0 + \frac{T_0}{\phi - \gamma^{\text{pre}}} \right) + (\phi - \gamma) \frac{\partial}{\partial\tau_i} \left( z_0 + \frac{T_0}{\phi - \gamma^{\text{pre}}} \right). \quad (23)$$

In (22), tax changes will indirectly affect  $G_0$  through their effect on  $c_0$ , and directly through the change in the growth rate of private consumption and through the effect on  $\beta$ . The second term in (22) is always positive as  $\partial\gamma/\partial\tau_i < 0$ ; a higher growth rate of private consumption from tax cuts makes Laffer effects more difficult to achieve. The third term comes from the impact of a tax change on the economy-wide return to capital. A change in  $\beta$ , through a change in compositional distortions that affects output in all periods, changes the present value of given flows of lifetime private and government consumption and thereby the scope for Laffer effects. Expression (23) is the standard consumption response in period-zero consumption through income and substitution effects (first term) and wealth effects of transfers (second term). The wealth effect of transfers plays a crucial role in the possibility to get Laffer effects<sup>20</sup>.

We sum up the effects from (22) and (23) and rewrite.  $\partial G_0/\partial\tau_i$  is a sum of the growth effects ( $\partial\gamma/\partial\tau_i$ ) on the two different present values of consumption and the effects through the different returns to capital. The condition to get a definition 1 Laffer effect,  $\partial G_0/\partial\tau_i < 0$ , becomes

$$\frac{\partial G_0}{\partial\tau_i} = \frac{\partial\gamma}{\partial\tau_i} \left( \frac{c_0}{\phi - \gamma} - \frac{c_0}{\beta - \gamma} \right) + \left( \frac{\partial\beta}{\partial\tau_i} - \frac{\partial\phi}{\partial\tau_i} \right) \left( \frac{c_0}{\phi - \gamma} - \frac{T_0}{\phi - \gamma} \right) < 0. \quad (24)$$

Using relationships (18) and (19) between changes in  $\gamma$ ,  $\beta$  and  $\phi$  gives

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<sup>19</sup>We differentiate with respect to a tax and then evaluate the derivative in the pre tax-cut point.

<sup>20</sup>From 23,  $c_0$  is affected through the change in the portion of lifetime income consumed in the first period  $(\phi - \gamma)$  and through the change in valuation of lifetime transfers  $T_0/(\phi - \gamma^{\text{pre}})$ . If the intertemporal elasticity of substitution  $\theta^{-1}$  is less than unity, the income effect dominates the substitution effect and the first term is negative. The second term is the wealth effect and is always positive. It will act to reduce period-zero consumption when taxes are reduced because future transfers are worth less as a result of the tax cut. The wealth effect from transfers must be sufficiently large, i.e. the higher  $\partial c_0/\partial\tau_i$ , the more likely is a Laffer effect.

$$\frac{\partial G_0}{\partial \tau_i} = \left( \frac{1}{\phi - \gamma} \right) \frac{\partial \phi}{\partial \tau_i} \left( \frac{1}{\theta} \frac{\beta - \phi}{\beta - \gamma} c_0 + (\Gamma_i - 1)(c_0 - T_0) \right) < 0. \quad (25)$$

From (25), because  $\partial \phi / \partial \tau_i < 0$  and  $\phi - \gamma > 0$ , there are two possible cases giving a Laffer effect:

**Proposition 1** *There is a Laffer effect,  $\partial G_0 / \partial \tau_i < 0$ , in the sense of definition (1), where  $i = k$  or  $h$ , if*

$$\frac{T_0}{c_0} > 1 - \frac{1}{1 - \Gamma_i} \frac{\beta - \phi}{\beta - \gamma} \quad \text{or if} \quad \Gamma_i > 1.$$

The first part of proposition 1 is written to stress the importance of transfers and the wealth effect that results from tax cuts. It simplifies to the case of the AK-model when taxes are equal, i.e. when  $\Gamma_i = 0$ , meaning that there is only a dynamic and no compositional margin. The proposition then tells us how large a share of consumption that should be transfer-financed to get a *dynamic* Laffer effect<sup>21</sup>.

We postpone the discussion of the criterion  $\Gamma_i > 1$ , proceed to the proposition regarding definition 2 Laffer effects and then interpret the results.

#### 4.4 Laffer effect according to definition 2

With government transfers and consumption following the (higher) growth rate of private consumption after a tax cut, the present value budget and resource constraints, (14) and (16), simplify to become

$$c_0 = z_0 (\phi - \gamma) + T_0 \quad (26)$$

$$G_0 = z_0 (\beta - \gamma) - c_0. \quad (27)$$

The condition to get a Laffer effect is, once more,  $\partial G_0 / \partial \tau_i < 0$ . Differentiation of  $G_0$  with respect to any tax gives

$$\frac{\partial G_0}{\partial \tau_i} = z_0 \left( \frac{\partial \beta}{\partial \tau_i} - \frac{\partial \phi}{\partial \tau_i} \right) = z_0 \frac{\partial \phi}{\partial \tau_i} (\Gamma_i - 1). \quad (28)$$

Because  $\partial \phi / \partial \tau_i < 0$ , we arrive at the following proposition:

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<sup>21</sup>See Agell and Persson (2001) for a full discussion.



**Proposition 2** *There is a Laffer effect,  $\partial G_0/\partial \tau_i < 0$ , in the sense of definition (2) if  $\Gamma_i > 1$ .*

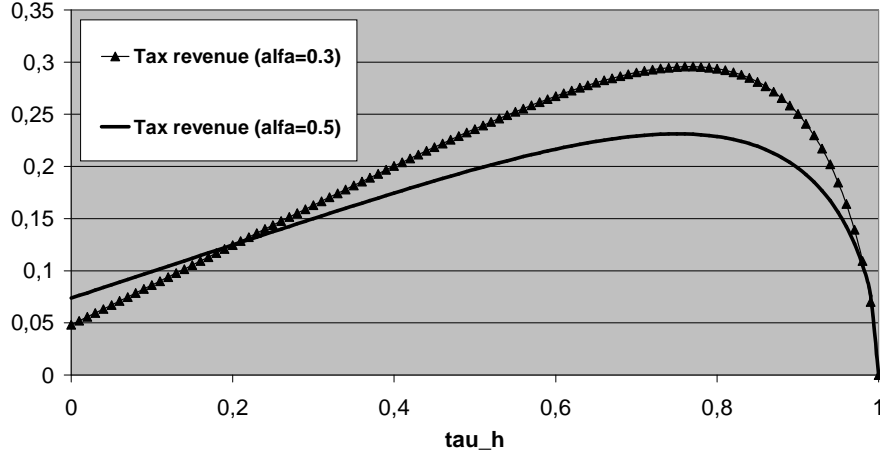
## 4.5 Interpretation of Laffer effects

As expected, definition 1 Laffer effects are the easiest to obtain. We only require government spending to grow at the old growth rate, whereas for definition 2 government spending should grow at the new higher growth rate that follows from a tax cut. We see that a positive  $\Gamma_i$  in the first part of proposition 1 reduces the requirement on the transfer/consumption ratio in order to obtain a Laffer effect. This is because a tax cut reduces the compositional distortion, thus helping the self-financing of a tax cut. The second part of proposition 1 is the same as proposition 2 and we now analyze the requirement for a definition 2 effect,  $\Gamma_i > 1$ , in more detail.

If we combine the intertemporal constraints (26) and (27), we get  $T_0 + G_0 = (\beta - \phi) z_0$ . Because  $(\beta - \phi) = \beta \tau_{avg}$  (from 11), this is nothing but the time-zero budget constraint of the government from (17). Since all variables grow at the same rate, the dynamic constraint collapses to the static government budget constraint. The reason is that with the assumptions on  $c$ ,  $T$  and  $G$  growing at the same growth rate, for the optimal solution also total capital  $z$  will grow at the same rate and no bonds will ever be issued. That is, if we are in a condition to get a dynamic Laffer effect,  $\Gamma_i > 1$ , no bonds are needed;  $G_0$  will be residually determined to fulfill the present value (and static) resource constraint. If we are not in a condition to get a dynamic Laffer effect,  $\Gamma_i < 1$ , if bonds were to be issued they could not be recovered (we would violate a transversality condition) and there is no way for  $G_0$  complying with  $\partial G_0/\partial \tau_i < 0$  to fulfill the present value resource constraint.

If we maximize static government revenue  $\beta \tau_{avg}$  keeping one tax constant (say  $\tau_k$ ), we get the condition that  $\tau_h$  should fulfill  $\Gamma_h = 1$  for maximum revenue. For  $\Gamma_h > 1$ , we are below maximum tax revenue. Therefore, the condition for definition 2 Laffer effects is the same as a static government revenue maximization problem. If  $\tau_h$  is beyond the point where  $\Gamma_h = 1$ , we are at the wrong side of the  $\tau_h$ -Laffer curve (graph below) and can hence increase revenue by reducing  $\tau_h$ . From the graph, we see that if the physical capital tax stands at  $\tau_k = 0.3$ , a human capital tax above 0.75 would be needed to get a definition 2 Laffer effect. As seen in the graph, this result is not very sensitive to the value of  $\alpha$ .

### Tax revenue for a given physical capital tax



**Figure 1.** The  $\tau_h$ -Laffer curve for two different values of  $\alpha$  when  $\tau_k = 0.3$ . (The shape of these curves does not depend on  $A$  which is set to 1 in this graph)

The analysis above tells us that there are two sources for Laffer effects, *compositional* and *dynamic*. In our model, where there are no transitional dynamics but immediate adjustment in the  $h/k$ -ratio, the compositional effect is static. As a result of a tax change, the agent immediately reoptimizes the  $h/k$ -ratio according to (8) and there is an immediate adjustment in the returns to capital and the consumption growth rate. Production  $\beta z$  will "jump" through the discrete adjustment in  $\beta$  and more resources are made available in all periods. This compositional effect makes it easier for the government to maintain spending and it is possible to get a definition 2 Laffer effect. The *dynamic* Laffer effect is captured by definition 1. Here, because of a less stringent requirement on spending and an increased growth rate of consumption and the capital stock, there is a true dynamic effect of tax cuts.

The interpretation of Laffer effects as compositional and dynamic is likely to carry over to the more general two-sector model with separate production functions for physical and human capital. Our model is a special case of this two-sector model; we have assumed one production function for both physical and human capital and immediate adjustment in the stocks of  $h$  and  $k$ . These assumptions have allowed us to separate compositional from dynamic effects. In the general model, along a balanced growth path, the  $h/k$ -ratio will also be constant. A tax change will result in a period of transition where the ratio - or composition - readjusts to the new tax rates. A tax cut in these models also generates the growth effect, which is the source of the dynamic Laffer effects.

We state a final proposition regarding Laffer effects and then proceed to

studying what the introduction of a labor/leisure choice implies for the analysis of Laffer effects.

#### 4.6 Laffer effect according to definition 3

With definition 3, total government spending should increase as a *fraction* of GDP<sup>22</sup>. It is straightforward to show that this can never be possible. Rewrite (27) using (26) to get

$$G_0 = z_0 (\beta - \phi) - T_0 = \beta \tau_{avg} z_0 - T_0.$$

Division by GDP,  $\beta z_0$ , gives

$$\frac{G_0}{GDP} = \tau_{avg} - \frac{T_0}{GDP}.$$

We are interested in the sign of  $\partial(G_0/GDP)/\partial\tau_i$ . If we reduce either tax rate, the factor  $\tau_{avg}$  will decrease. If the  $T_0/GDP$ -ratio is to remain intact, the left-hand side must then decrease as a result of the tax cut, i.e.  $\partial(G_0/GDP)/\partial\tau_i > 0$ . We are thus in a case where there are no dynamic or compositional margins from which resources for increased government consumption can be generated and we get the following result;

**Proposition 3** *There can never be a Laffer effect,  $\partial(G_0/GDP)/\partial\tau_i < 0$ , in the sense of definition (3).*

### 5 Adding leisure to the model

So far in this paper we have seen how capital taxation in general and an uneven taxation of factors of production in particular affect the scope for self-financing tax cuts. When a tax is reduced, dynamic and compositional margins are affected and there may be Laffer effects. There is symmetry between changes in  $\tau_k$  and  $\tau_h$ . In this section, I extend the model by introducing a labor/leisure choice and leisure in the agent's utility function. I follow most of the literature and model leisure as "raw-time", where human capital and leisure are bundled together and the human capital effectively supplied for production is  $h(1 - l)$

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<sup>22</sup>Increasing the total government spending ( $G + T$ ) to GDP ratio is equivalent to asking whether  $G$  can increase as a fraction of the new GDP, letting  $T$  increase to exactly preserve its GDP ratio.

rather than  $h$ . Here,  $(1 - l)$  is the fraction of the unitary time endowment used for labor and  $l$  is leisure time<sup>23</sup>.

Before presenting the extended model, we can say something about what results we expect. First, we should expect the scope for Laffer effects to increase because we have a new margin of adjustment. As we will see, the growth rate in this model will be increasing in the level of labor time. Therefore, a tax cut that increases labor time adds a new dynamic margin which is indeed a new source of a *dynamic* Laffer effect. Second, there is also a new compositional effect. If labor time and therefore production increase in response to a tax cut, this also opens up for Laffer effects (although we also need to consider the general equilibrium response in the  $h/k$ -ratio).

We should also expect the introduction of leisure to break the symmetry between the two taxes. In particular, the agent will be faced with an intratemporal allocation decision between consumption and leisure. This decision will be directly affected by the tax on human capital, whereas the physical capital tax will only have indirect effects on the consumption/leisure decision.

A limitation in the analysis is that the method to integrate the budget and resource constraints will no longer be easily applicable for definition 1 Laffer effects. This is due to the fact that no constant-leisure level will exist other than in the long run when we have variables growing at different growth rates. An analytical condition for definition 1 Laffer effects with leisure will therefore not be provided. We can, however, get the intuition for definition 1 Laffer effects, discussing how a tax cut has affected the growth rate through the leisure level. For definition 2 Laffer effects, we will get a solution where leisure jumps from one constant level to another as taxes change. This means that consumption, capital stocks, government consumption and government transfers can all grow at the same rate and we can analyze the compositional effect of having introduced leisure<sup>24, 25</sup>.

## 5.1 The model with leisure

I limit the model description here to what has changed from above. A fraction  $l$  of the agent's unitary time endowment will be removed from production. The

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<sup>23</sup>See Milesi-Ferretti and Roubini (1998a) for a discussion of different specifications of leisure. We also refer to these authors for a full discussion of the problem set-up and first-order conditions. Our model is a special case of their model, the case when the production functions for physical and human capital are the same.

<sup>24</sup>We use a utility function that is consistent with a steady state with a constant leisure level as derived by King et al. (1988).

<sup>25</sup>The definitions of Laffer effects remain the same. For definition 2 effects, this still means that if there is such an effect, it will be compositional in nature. Leisure in the model may change the way the growth rate responds to tax cuts. This change in growth rate also applies to government spending, however.

remaining part of the time endowment,  $(1 - l)$ , will be used in production so that effective human capital supplied in production is  $h(1 - l)$  and  $w = \partial f / \partial (h(1 - l))$  will be the return to effective human capital. We can proceed with the model setup from above, but we need to replace  $h$  with  $h(1 - l)$  in the production function and in the budget constraint<sup>26</sup>. We continue suppressing time indices on the variables and, in order to not introduce additional confounding notation, we use the same symbols  $w, r, \gamma, \phi, \beta$  as above. As an example,  $w$  is still the return to human capital but its expression will be slightly different from above because of the introduction of leisure in the model.

The arbitrage condition (1) now becomes

$$r(1 - \tau_k) = w(1 - l)(1 - \tau_h). \quad (29)$$

From this condition, we derive the  $h/k$ -ratio, which will still be  $h/k = \Omega$ . It is unaffected by the introduction of leisure, but we note that part of the human capital stock is no longer deployed<sup>27</sup>. Knowing  $h/k$ , we can derive the expressions for the private return  $\phi$  and the economy-wide return  $\beta$ . These will look as in the no-leisure case, (2) and (11), but will now include a leisure component  $(1 - l)^{1-\alpha}$ :

$$\begin{aligned} \phi &\equiv A\alpha^\alpha(1 - \alpha)^{1-\alpha}(1 - \tau_k)^\alpha(1 - \tau_h)^{1-\alpha}(1 - l)^{1-\alpha}. \\ \beta &= \frac{\phi}{1 - \tau_{avg}}. \end{aligned}$$

From these expressions, we see that the introduction of leisure has affected the returns to capital in the economy. The fact that not all human capital is deployed in production has a negative effect on the return to capital and, as we shall see, the growth rate. It follows that changes in leisure, induced by tax cuts, will affect the scope for Laffer effects. In particular, it seems likely that decreases in the leisure level from tax cuts,  $\partial l / \partial \tau_i > 0$ , will act as a new margin that increases the scope for self-financing tax cuts both through more human capital deployed in production and through a higher growth rate<sup>28</sup>. This growth effect is in addition to the positive effect on the growth rate from the tax cut

<sup>26</sup>When we solved the representative agent's problem in the non-leisure section, we first derived the non-arbitrage condition between  $k$  and  $h$  and then worked with the state variable  $W$  (or equivalently,  $z$  and  $b$ ) in the optimization set-up.

With leisure,  $h$  is now replaced by  $h(1 - l)$ . There are first-order effects of changes in  $l$  and we need to explicitly express the return to capital in the budget constraint, i.e.  $hw(1 - l)(1 - \tau_h) + kr(1 - \tau_k)$ .

<sup>27</sup>A fraction  $l$  of human capital  $h$  is no longer productive. The return on effective human capital  $h(1 - l)$  has increased and the return on  $k$  has decreased. The  $h/k$ -ratio that satisfies condition (29) remains intact. The fraction of human capital used in production, i.e.  $h(1 - l)/k$ , has decreased, though, which is what we should expect.

<sup>28</sup>The total (general equilibrium) effect also needs to take the change in the  $h/k$ -ratio into account.

itself and thus constitutes an additional margin for definition 1, or *dynamic*, Laffer effects.

Although no explicit expression for definition 1 Laffer effects and no proposition corresponding to proposition 1 are presented, it is thus likely that the requirement to have a definition 1 Laffer effect is less stringent than in the no-leisure case.

We now define a utility function and proceed to analytically characterizing definition 2 Laffer effects. The utility function is additively separable in the private goods  $(c, l)$  and government consumption  $(G)$ ,  $U(c, l, G) = u(c, l) + v(G)$ . We follow Devereux and Love (1994) and Novales and Ruiz (2002) and use the following utility function:

$$u(c, l) = \frac{(c^\eta l^{1-\eta})^{1-\theta}}{1-\theta}.$$

King et al. (1988) have shown this utility function to be consistent with a balanced growth path with constant leisure. The parameter  $\eta$  indicates the relative preference for consumption. The first-order conditions with respect to consumption and leisure imply that the marginal rate of substitution should equate the relative price:

$$\frac{u_l}{u_c} = wh(1 - \tau_h). \quad (30)$$

The human capital tax has a direct effect on this trade-off through its direct effect on the relative price of leisure. Both  $\tau_h$  and  $\tau_k$  also indirectly affect the trade-off through changes in  $w$  and  $h$ . We can see the full tax-dependence in the consumption/leisure trade-off by rewriting (30) using the utility function and the general equilibrium expressions for  $w$  and  $h$ :

$$\frac{c}{l(1-l)^{-\alpha}} = \frac{\xi(1-\tau_h)^{2-\alpha}(1-\tau_k)^\alpha}{\alpha(1-\tau_k) + (1-\alpha)(1-\tau_h)}. \quad (31)$$

Here,  $\xi$  is a function of the capital stock  $z$  and the non-tax parameters of the problem<sup>29</sup>. The right-hand side is unambiguously decreasing in  $\tau_h$  and it is also decreasing in  $\tau_k$  if  $\tau_k > \tau_h$  which is the case when we study reductions in  $\tau_k$ . Moreover, it can be shown that the size of the effect on the intratemporal margin from tax cuts is larger for changes in  $\tau_h$  than in  $\tau_k$  (because of the direct effect

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<sup>29</sup>  $\xi = Az \frac{\eta}{1-\eta} \alpha^\alpha (1-\alpha)^{2-\alpha}$ .

of  $\tau_h$ ). From these considerations, we can conclude that the consumption-to-leisure ratio increases for tax cuts that are such that the highest tax is reduced and the ratio increases more for reductions in  $\tau_h$  than for reductions in  $\tau_k$ . We are thus lead to expect the largest jump in the leisure level from reductions in  $\tau_h$  and possibly a larger scope for Laffer effects if in a situation with  $\tau_h > \tau_k$  than from reducing  $\tau_k$  when  $\tau_k > \tau_h$ .

We still need to derive the intertemporal conditions, both to get a second relationship between  $c$  and  $l$  from the consumption rule and to get the present value resource constraint from which we study Laffer effects. The Euler equation is once again derived from first-order conditions similar to (5) and (6), where the utility function and  $\phi$  now also include leisure. With  $\sigma \equiv (1 - \eta + \eta\theta)^{-1}$ , the Euler equation becomes

$$\frac{\dot{c}}{c} - \frac{\dot{l}}{l} \sigma (1 - \theta) (1 - \eta) = \sigma (\phi - \rho). \quad (32)$$

With constant leisure, the Euler equation reduces to  $\gamma \equiv \frac{\dot{c}}{c} = \sigma (\phi - \rho)$  and the method of integrating the budget and resource constraints remains valid. The Euler equation looks as before but the "intertemporal elasticity of substitution"  $\sigma$  now equals  $(1 - \eta + \eta\theta)^{-1}$ . It collapses to  $\theta^{-1}$  when the representative agent only values consumption ( $\eta = 1$ ). With the Euler equation derived, we can proceed to derive the conditions for definition 2 Laffer effects using the same method as in the section without leisure.

## 5.2 Laffer effect according to definition 2 – with leisure

The expressions for the consumption rule and the present value resource constraint from (26) and (27) remain valid; they maintain their simple form because any adjustment in the growth rate also applies to government transfers  $T$  and government consumption  $G$ . The only difference is the factor  $(1 - l)^{1-\alpha}$  now present in  $\gamma$ ,  $\phi$  and  $\beta$ . We repeat these expressions for convenience:

$$c_0 = z_0 (\phi - \gamma) + T_0 \quad (33)$$

$$G_0 = z_0 (\beta - \gamma) - c_0. \quad (34)$$

To get an explicit expression for Laffer effects, we first need to solve for  $c$  and  $l$  from the intratemporal (equation 31) and intertemporal (equation 33)

relationships. Implicitly, however, we can follow the procedure from the non-leisure section in that we differentiate (34) and (33) to obtain the condition for definition 2 Laffer effects. That is, we calculate the derivative  $\partial G_0 / \partial \tau_i = z_0 (\partial \beta / \partial \tau_i - \partial \phi / \partial \tau_i)$  as in (28), without first solving for the leisure level, explicitly recognizing that  $\beta$  and  $\phi$  contain the factor  $(1 - l)^{1-\alpha}$  where  $l$  is tax-dependent. This means that the proposition will contain the leisure level itself, a variable for which we have not yet solved. It turns out that we can draw qualitative conclusions from the analysis if we manage to determine how leisure reacts to tax changes, i.e. the sign of  $\partial l / \partial \tau_i$ .

From the criterion for a Laffer effect,  $\partial G_0 / \partial \tau_i < 0$ , we get the following proposition, using the auxiliary positive parameter  $\Psi_i$ <sup>30</sup>:

**Proposition 4** *In the case with leisure, there is a Laffer effect,  $\partial G_0 / \partial \tau_i < 0$ , in the sense of definition (2), if  $\Gamma_i > 1 - \Psi_i \frac{\partial l / \partial \tau_i}{1 - l}$ .*

The difference between propositions (4) and (2) is the term  $\Psi_i \partial l / \partial \tau_i / (1 - l)$ . The scope for a definition 2 Laffer effect is larger or smaller in the model with leisure depending on the sign of the leisure derivative with respect to the tax rate. A positive tax derivative,  $\partial l / \partial \tau_i > 0$ , produces a larger change in  $\beta$  than in  $\phi$ , as compared to the non-leisure case, thereby facilitating the desired  $\partial \beta / \partial \tau_i - \partial \phi / \partial \tau_i < 0$ . The opposite holds for a negative tax derivative,  $\partial l / \partial \tau_i < 0$ . The earlier requirement that the compositional distortion must be such that  $\Gamma_i > 1$  in order to get a definition 2 Laffer effect is thus "relaxed" when there is leisure in the model (if  $\partial l / \partial \tau_i > 0$ ) because a tax cut also helps through the leisure margin.

### 5.3 Interpretation of Laffer effects, with leisure

The introduction of leisure into the model has opened up a new channel for dynamic as well as compositional Laffer effects. Although we do not study definition 1 Laffer effects in this section, due to analytical complexity, we see that the labor/leisure level affects the growth rate. The private return  $\phi \equiv A\alpha^\alpha(1 - \alpha)^{1-\alpha}(1 - \tau_k)^\alpha(1 - \tau_h)^{1-\alpha}(1 - l)^{1-\alpha}$  and therefore the growth rate,  $\gamma = \sigma(\phi - \rho)$ , will increase as either tax  $\tau_i$  is reduced. If  $\partial l / \partial \tau_i > 0$ , there is an additional boost to the growth rate. This is a new dynamic effect. It is likely to make it easier for the government to maintain its spending path according to Laffer effect definition 1, as compared to the no-leisure case.

As for definition 2 effects, we once again get an expression for the compositional distortion required for there to be a Laffer effect from tax cuts. If

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<sup>30</sup>  $\Psi_k = \frac{1 - \alpha}{\alpha} \frac{1 - \tau_k}{1 - \tau_{avg}} \tau_{avg}$  and  $\Psi_h = \frac{1 - \tau_h}{1 - \tau_{avg}} \tau_{avg}$

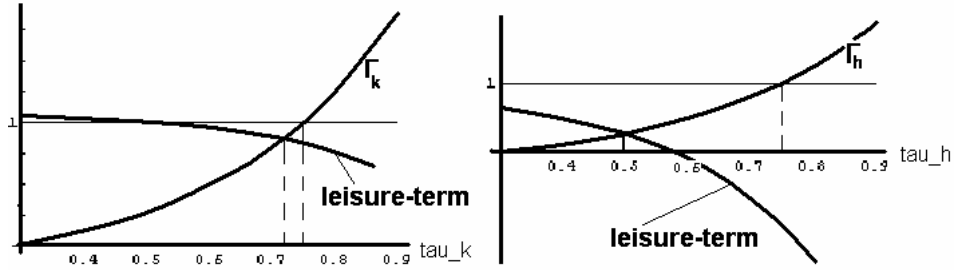


$\partial l / \partial \tau_i > 0$ , which is likely to be the case at least for the human capital tax, the agent will work more and production will increase. We start out in a situation with production  $\beta z$  where  $\beta$  is increasing in labor time  $(1 - l)$ . This effect comes in addition to any effect through the  $h/k$ -ratio which, as previously, is determined by  $\Omega$ . Thus, we have a new compositional margin that may increase production and therefore be a source of definition 2 Laffer effects. This is what is captured by proposition (4) above.

In the appendix, we solve for the leisure level from (33) and (31). This can only be done for certain values of  $\alpha$  and the analysis is therefore suggestive at most. We show that the leisure-to-tax derivatives, comparing situations such as  $\partial l / \partial \tau_h / (1 - l)$  when  $(\tau_h, \tau_k) = (0.5, 0.3)$  with  $\partial l / \partial \tau_k / (1 - l)$  when  $(\tau_h, \tau_k) = (0.3, 0.5)$ , are likely to obey the following conditions:

$$\frac{\partial l / \partial \tau_h}{1 - l} \gg \frac{\partial l / \partial \tau_k}{1 - l} \text{ and } \frac{\partial l / \partial \tau_h}{1 - l} > 0.$$

Therefore, changes in the human capital tax are likely to be more effective in creating Laffer effects. This applies to both the dynamic and the compositional part of the effect. Figure 2 below illustrates the left- and right-hand sides of proposition (4) where the  $\Gamma_i$ -term, i.e. the left-hand side, should be larger than the leisure term  $1 - \Psi_i \partial l / \partial \tau_i / (1 - l)$  for there to be a definition 2 Laffer effect. The left graph is for reductions in  $\tau_k$  and the right graph is for reductions in  $\tau_h$ . In the left graph, we fix  $\tau_h$  at 30% and vary  $\tau_k$  above this rate. In the right graph, we fix  $\tau_k$  at 30% and vary  $\tau_h$  above this rate<sup>31</sup>.



**Figure 2.** Requirements on  $\tau_k$  to get a Laffer effect when  $\tau_h$  is fixed at 30% (left graph) and requirements on  $\tau_h$  to get a Laffer effect when  $\tau_k$  is fixed at 30% (right graph). When leisure is in the model, the requirement on the tax rate to get a Laffer effect moves from the intersection of the  $\Gamma$ -term and "1" to the intersection of the  $\Gamma$ -term and the leisure-term.

The graphs show what level of tax rates is required for definition 2 Laffer effects, both in the model with and without leisure, when one tax rate is kept at 30%. The requirements are lower when leisure is endogenous. To the right of the

<sup>31</sup>Parameters are:  $\alpha = 0.5$ ,  $A = 0.1$ ,  $\theta = 2$ ,  $\eta = 0.7$ ,  $\rho = 0.02$  and half of government revenue goes to transfers.

$\Gamma$ -leisure intersection, we have a Laffer effect. This occurs at around  $\tau_k = 70\%$  in the left graph; above this level reducing  $\tau_k$  produces a Laffer effect. For the right graph, above  $\tau_h = 50\%$  we get a Laffer effect for reductions in  $\tau_h$ . In the model without leisure, we have a Laffer effect to the right of the intersection between the  $\Gamma$ -curve and "1". This is also illustrated in the graphs and occurs at higher tax rates (i.e. larger compositional distortions). The graphs also show that the effects of introducing leisure are larger for the human capital tax.

We conclude the leisure section by stating that Laffer effects according to definition 3 are not possible in the case with leisure. The reasoning from proposition 3 remains unchanged.

**Proposition 5** *In the case with leisure, there can never be a Laffer effect,  $\partial(G_0/GDP)/\partial\tau_i < 0$ , in the sense of definition (3).*

## 6 Discussion

In the general version of the two-sector model, there is a separate sector accumulating human capital. An endogenous labor/leisure choice is also standard in such models. As a result of tax cuts, a period of transitional dynamics follows, during which stocks of human and physical capital allocated to each sector readjust. There is also an increase in the growth rate. In this paper, I have made simplifying assumptions regarding technology and the transition phase in order to separate different effects arising from tax cuts.

First, there are adjustments along a dynamic margin from the tax cut itself – the growth rate increases. This opens up for *dynamic* Laffer effects if we assume that government spending grows at its pre tax cut growth rate. With endogenous leisure, the growth rate also increases because more human capital is deployed, this is a second source of a *dynamic* Laffer effect.

Second, there are adjustments along a *compositional* margin if we have more than one factor of production. Tax cuts change the human to capital equilibrium composition. This affects production and is a source of Laffer effects. The level of leisure also changes as a result of the tax cut. This also changes production and is a second source of a *compositional* Laffer effect.

Compositional and dynamic distortions need to be quite large in order to get a Laffer effect. Fixing one tax rate at 30%, the other tax rate must be above 70% if a reduction in this higher tax is to be self-financing (for one parametrization of the model). If leisure is in the model, this requirement goes down. Through the direct effect on the consumption/leisure trade-off, a human capital tax cut produces a direct effect which is not present for a physical capital tax cut. Therefore, human capital tax cuts are a more likely source of Laffer effects when there is leisure in the model. For one parametrization, fixing the physical capital tax rate at 30% requires the human capital tax rate to be above 50% to get a Laffer effect (instead of above 70% when leisure is not in the model).

## 7 References

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## 8 Appendix

### 8.1 Variables and parameters

$c$  is consumption

$l$  is leisure

$k$  is the agent's physical capital stock

$h$  is the agent's human capital stock

$z = k + h$  is the agent's total capital stock

$z_0 = k_0 + h_0$  is the agent's total initial capital stock

$b$  is the agent's government bond stock

$b_0 = 0$  is the agent's initial government bond stock

$W$  is the agent's total wealth

$W_0 = z_0$  is the agent's initial wealth

$T$  is the government lump-sum transfer to the agent

$G$  is government consumption

$r$  is the market rate of return on physical capital

$w$  is the market rate of return on human capital

$\rho$  is the rate of time preference of the agent

$\tau_k$  is the physical capital and government bond income tax

$\tau_h$  is the human capital income tax

$\theta^{-1}$  is the intertemporal elasticity of substitution in the no-leisure case

$\eta$  is the relative preference for consumption in the utility function

$(1 - \eta + \eta\theta)^{-1}$  is the intertemporal elasticity of substitution in the leisure case

$\alpha$  is the relative weight of physical capital in production ( $0 < \alpha < 1$ )

$A$  is a constant in the production function

$$\tau_{avg} = \alpha\tau_k + (1 - \alpha)\tau_h$$

$$\Omega = \frac{(1 - \alpha)(1 - \tau_h)}{\alpha(1 - \tau_k)}$$

$$\phi = A\alpha^\alpha(1 - \alpha)^{1-\alpha}(1 - \tau_k)^\alpha(1 - \tau_h)^{1-\alpha} \text{ in the no-leisure case and}$$

$$\phi = A\alpha^\alpha(1 - \alpha)^{1-\alpha}(1 - \tau_k)^\alpha(1 - \tau_h)^{1-\alpha}(1 - l)^{1-\alpha} \text{ in the leisure-case}$$

$$\gamma = \theta^{-1}(\phi - \rho) \text{ is the growth rate of consumption}$$

$$\beta = \frac{A\Omega^{1-\alpha}}{1 + \Omega} = \frac{\phi}{1 - \tau_{avg}} \text{ is the economy-wide return to capital}$$

$$\Gamma_k = \frac{(1 - \alpha)(\tau_k - \tau_h)}{(1 - \tau_{avg})^2} \text{ and } \Gamma_h = \frac{\alpha(\tau_h - \tau_k)}{(1 - \tau_{avg})^2}$$

$$\Psi_k = \frac{1 - \alpha}{\alpha} \frac{1 - \tau_k}{1 - \tau_{avg}} \tau_{avg} \text{ and } \Psi_h = \frac{1 - \tau_h}{1 - \tau_{avg}} \tau_{avg}$$

### 8.2 Relationship to the general two-sector model

The model in this paper uses two assumptions regarding the production function and reversibility of investments that require some motivation. In discussing these assumptions and their implications below we are very brief, and refer

details to the work by Milesi-Ferretti and Roubini (1998a, 1998b) and papers referenced therein<sup>32</sup>. Barro and Sala-i-Martin (1995) also discuss the model used here. For simplicity, I discuss the model assumptions in a setting without leisure. The first assumption is that output and human capital are produced with the same production function. Second, there are no restrictions on deinvestment in the stocks of  $H$  and  $K$ .

For the first assumption, consider the general model where output is produced with the following production function,  $Y = A_K (vK)^{\alpha_1} (uH)^{1-\alpha_1}$  and human capital is produced with  $\dot{H} = A_H ((1-v)K)^{\alpha_2} ((1-u)H)^{1-\alpha_2}$ . Physical capital input is divided according to  $vK$  used for final good production and  $(1-v)K$  for human capital production and there is a similar division of human capital input. As discussed at some length in Barro and Sala-i-Martin (1995), this model is difficult to analyze in its general version. If  $\alpha_1 = \alpha_2 \equiv \alpha$ , however, things simplify a lot. From the first-order conditions, we get that when  $\alpha_1 = \alpha_2$ , the marginal impact of increasing the fraction  $v$  has the same impact on final good production relative to human capital production as an additional unit of  $u$  has on output production relative to human capital production. Therefore,  $v = u$  so that irrespective of the global  $H/K$ -ratio, an equal fraction of each stock is deployed in final good production and the rest in human capital production. As a consequence, the relative price of human capital produced to final good produced is unaffected by the global  $H/K$ -ratio and it is also unaffected by  $v$  (because  $u$  adjusts to always be equal to  $v$ ). The relative price is therefore constant and being equal to  $p = A_K/A_H$ . Broad output can therefore be simplified to become  $Q \equiv Y + p\dot{H} = A_K (vK)^\alpha (vH)^{1-\alpha} + pA_H ((1-v)K)^\alpha ((1-v)H)^{1-\alpha} = A_K K^\alpha H^{1-\alpha}$ . This is the "broad" production function used in this paper.

The second assumption is that the model does not exhibit any transitional dynamics. The relative price of human to physical capital is constant and a unit of physical capital can be "deinvested" and instantaneously converted into a unit of human capital. Transitional dynamics could be included in the model by adding non-negativity constraints on capital accumulation. Outside the balanced growth  $H/K$ -ratio, the stock which is relatively abundant would remain constant for a finite time whereas the other stock would catch up until the equilibrium ratio is reached. Although there are interesting properties of this

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<sup>32</sup>Milesi-Ferretti and Roubini (1998a, 1998b) study growth responses from capital taxation in a catalogue of models using physical and human capital as factors of production and in which there is a different technology for producing human capital than for the final good. They use different versions of production functions in the two sectors; they separate the two cases where human capital is or is not a market activity and also elaborate on different leisure specifications.

In the terminology of Milesi-Ferretti and Roubini, our model falls under the category "raw-time" leisure, human capital is a market good and physical and human capital are produced with the same production function. The paper by Novales and Ruiz (2002), on the other hand, falls under the category "raw-time leisure", human capital is a non-market activity and human and physical capital are produced with different production functions.

transition, it is not the objective of this paper and it would considerably complicate the analysis of Laffer effects.

The assumption of  $\alpha_1 = \alpha_2$  means that human capital production is as intensive in physical capital input as is the production of output. A more realistic assumption is that human capital is relatively intensive in human capital input,  $\alpha_2 < \alpha_1$ . This is the Uzawa-Lucas model and in its extreme version  $\alpha_2 = 0$  such that  $\dot{H} = A_H(1-u)H$ . Milesi-Ferretti and Roubini (1998a), King and Rebelo (1990) and Rebelo (1991) all discuss this model with respect to capital taxation. Growth along the balanced growth path will be driven by human capital accumulation and in the case of a taxed human capital sector, the growth rate is a function of  $\tau_h$  but not of  $\tau_k$ . Changes in  $\tau_k$  will cause changes in the  $H/K$ -ratio used in producing output, but will not affect the growth rate. For Laffer effects, only the human capital tax has a dynamic effect. Presumably; in the intermediate model with  $0 < \alpha_2 < \alpha_1$ , the lower is  $\alpha_2$ , the more important is  $\tau_h$  for the possibility to obtain Laffer effects, although  $\tau_k$  plays some role. In the case with  $\alpha_1 = \alpha_2$ , there is complete symmetry between the two taxes (absent leisure).

### 8.3 Analytical solution with leisure

By combining the intertemporal relation (33) with the time-zero version of the intratemporal constraint (31), we eliminate consumption and get one equation in one unknown<sup>33</sup>, the time-zero leisure level  $l_0$ :

$$l_0(C_1 + C_2) = C_2 + C_3(1 - l_0)^\alpha.$$

This equation cannot be solved in the general case but we solve it for the special cases of  $\alpha = 1/2$  and (the more realistic)  $\alpha = 1/3$ . The leisure level is a non-trivial function of initial transfers  $T_0$  to initial capital  $z_0$  and all parameters including the tax rates<sup>34</sup>,  $l(T_0/z_0, A, \theta, \eta, \rho, \tau_h, \tau_k)$ . It can be differentiated, however, so that we get analytical expressions for the tax derivatives  $\partial l / \partial \tau_h$

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<sup>33</sup>The three constants are  

$$C_1 = \frac{\eta}{1-\eta} \frac{Az_0\alpha^\alpha(1-\alpha)^{2-\alpha}(1-\tau_h)^{2-\alpha}(1-\tau_k)^\alpha}{\alpha(1-\tau_k) + (1-\alpha)(1-\tau_h)}$$

$$C_2 = A \left(1 - (1 + \eta\theta - \eta)^{-1}\right) \alpha^\alpha(1-\alpha)^{1-\alpha}(1-\tau_k)^\alpha(1-\tau_h)^{1-\alpha}$$

$$C_3 = \frac{T_0}{z_0} + \rho(1 + \eta\theta - \eta)^{-1}$$

<sup>34</sup>For the case of  $\alpha = 1/2$ ;

$$l = \frac{C_2}{C_1 + C_2} - \frac{C_3^2}{2(C_1 + C_2)^2} + \frac{(C_3^4 + 4C_1^2 + 4C_1C_2)^{1/2}}{2(C_1 + C_2)^2}$$

Because  $l$  is constant, we drop the subindex 0.



and  $\partial l / \partial \tau_k$ . The Mathematica software is used for these calculations as well as when solving for the case of  $\alpha = 1/3$ .

When we calculate  $\partial l / \partial \tau_h$  we set  $\tau_h > \tau_k$  and vice versa when computing  $\partial l / \partial \tau_k$ . This is done because we are interested in tax cuts where we decrease the highest tax. As an example, if we use  $\tau_h=0.5$  and  $\tau_k=0.3$  to evaluate  $l$  and  $\partial l / \partial \tau_h$  to get the term  $\partial l / \partial \tau_h / (1 - l)$ , we compare this tax cut to a situation where we use  $\tau_k=0.5$  and  $\tau_h=0.3$  to evaluate  $l$  and  $\partial l / \partial \tau_k$  to get the term  $\partial l / \partial \tau_k / (1 - l)$ .

As discussed in the main text, the intratemporal condition (31) is such that an increase in either tax makes the agent substitute consumption for leisure. In the intertemporal relationship (33), a tax cut will reduce the consumption level for a constant leisure level (for  $\sigma < 1$ ). The full intertemporal effect also depends on the reaction of the leisure level to the tax cut, so we cannot draw a clear conclusion of the sign. However, the relationship between  $c$  and  $l$  in (33) is symmetric in the tax rates, so that any difference in the response in  $l$  between changes in  $\tau_h$  and  $\tau_k$  will be due to the intratemporal constraint.

Having taken derivatives, we derive comparative statics of  $l$  and  $\partial l / \partial \tau_i$  with respect to the variables  $(T_0/z_0, A, \theta, \eta, \rho)$  varying one parameter at a time<sup>35</sup>. The main results can be summarized as follows

- As expected, the level of leisure  $l$  is decreasing in the preference for consumption  $\eta$ . It is also decreasing in  $A$  so that the substitution effect dominates and the agent works more when  $A$  increases.
- From the intertemporal constraint (33), period zero consumption is increasing in the time preference factor  $\rho$  and in  $T_0/z_0$ . The intratemporal  $c/l$ -ratio is not affected, however, meaning that leisure must also be increasing in  $\rho$  and  $T_0/z_0$ .
- The only ambiguous comparative static is with respect to  $\theta$  as it will depend on the preference parameter  $\eta$ . For low values of  $\eta$ , the leisure level is decreasing in  $\theta$  and for high values of  $\eta$ , leisure is increasing in  $\theta$ <sup>36</sup>.

For the parametrizations of  $(\tau_k, \tau_h, T_0/z_0, A, \theta, \eta, \rho)$  and for both values of  $\alpha$ , we get the result that

<sup>35</sup>Typical parameter values are  $0.1 \leq A \leq 0.5$ ,  $1.1 \leq \theta \leq 5$ ,  $0.1 \leq \eta \leq 0.9$ ,  $0 \leq \rho \leq 0.05$ . In doing the comparative statics,  $T_0/z_0$  can be no higher than  $\tau_{avg}\beta$  which is the case when all government revenue is used for transfers. Choosing  $T_0/z_0$  thus amounts to choosing the ratio of government revenue that is used for transfers and we let this ratio vary between 0.1 and 0.9. Furthermore, we set the lowest tax rate at 0.3 and increase the other tax up to 0.9. We always check that there is positive growth and that the transversality condition holds.

<sup>36</sup>From  $\sigma = (1 - \eta + \eta\theta)^{-1}$ , we see that low values of  $\eta$  make  $\sigma$  very close to unity, substitution and income effects cancel out, and the fraction of lifetime wealth that is consumed in period zero is  $(\phi - \gamma) \approx \sigma\rho$ . The effect of a change in  $\sigma$  is then mainly to make the effective time discount factor larger, increasing consumption in period zero and also leisure (through the intratemporal constraint that has not changed). For large values of  $\eta$ , even small increases in  $\theta$  will lead to large changes in  $\sigma$  so that the fraction of lifetime wealth consumed in period zero now decreases in  $\sigma$ . Period zero consumption will then decrease in  $\sigma$  and so will the leisure level. Because  $\sigma$  is inversely proportional to  $\theta$ , the comparative statics with respect to  $\theta$  follow.

$$\frac{\partial l / \partial \tau_h}{1-l} \gg \frac{\partial l / \partial \tau_k}{1-l} \text{ and } \frac{\partial l / \partial \tau_h}{1-l} > 0.$$

In most cases also  $\partial l / \partial \tau_k / (1-l)$  is positive. At least for the parametrizations used we can thus say that, in order to get a Laffer effect, a human capital tax cut helps a great deal more than a physical capital tax cut. The model thus delivers what the basic intuition tells us, i.e. that the direct effect on the price of leisure creates a larger effect on the leisure level from human capital tax cuts than from physical capital tax cuts. The lower level of leisure delivered by the tax cut then helps producing a definition 2 Laffer effect according to proposition 4.